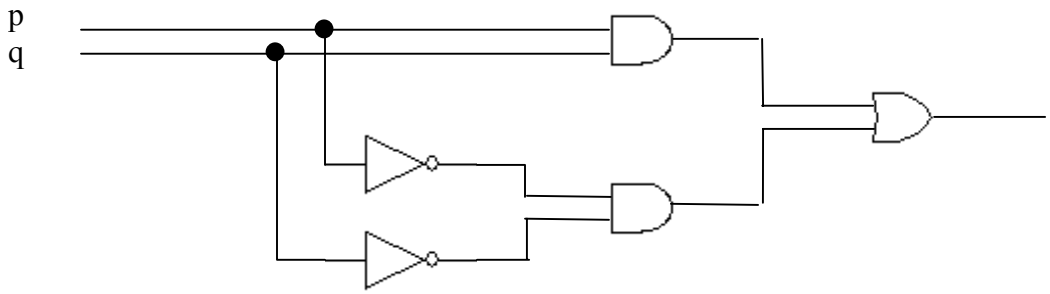
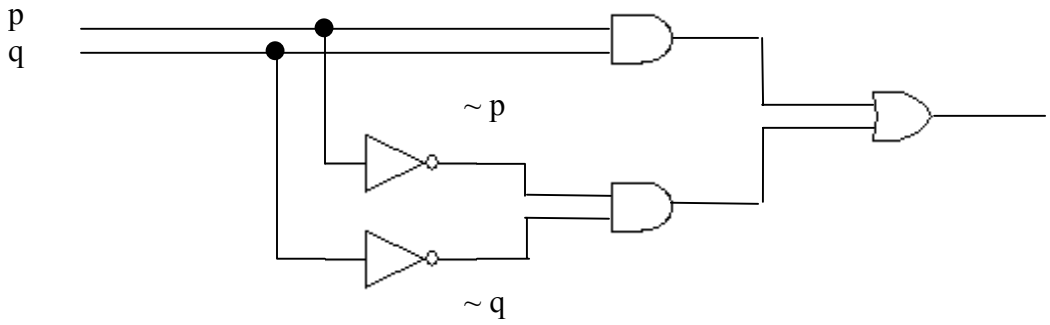
Logic Circuits and Logical Formulas – Working Systematically
(Revised Sept 24, 2004)

Difficulties that some students have in translating between circuit diagrams and formulas of propositional logic appear to result from "winging it" rather than proceeding systematically. The following examples – diagram to formula and then formula to diagram – illustrate a systematic approach.

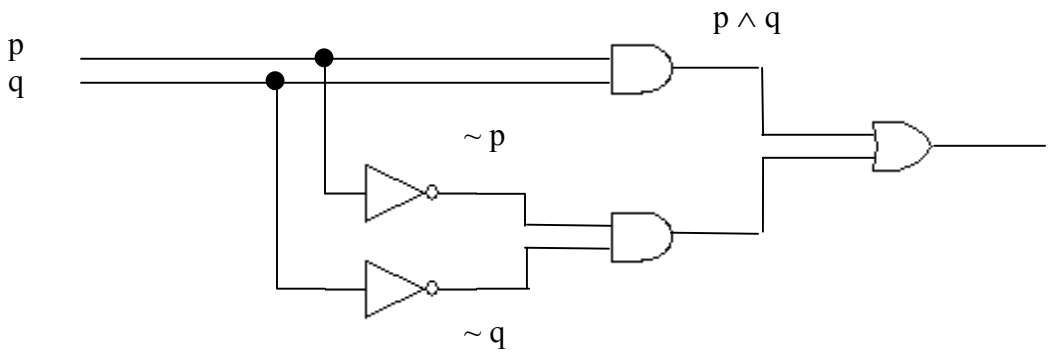
Consider the diagram below, where there are two inputs labeled p and q. Develop a formula.



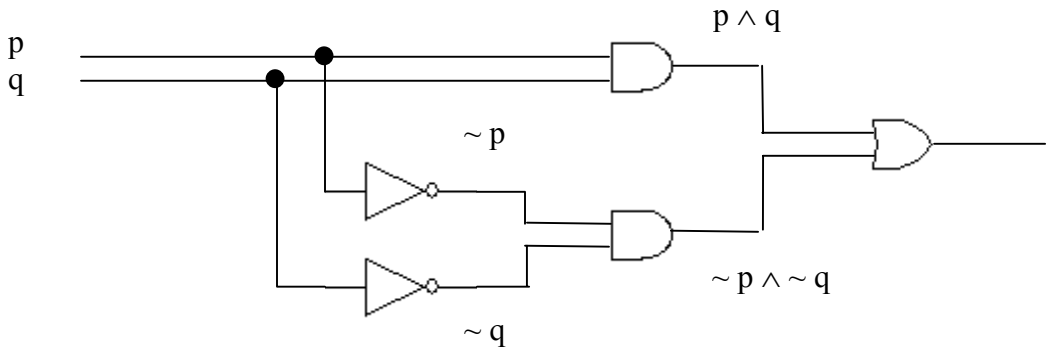
We work from left to right, labeling the output of each gate with a formula. First, look at the inverters. These form negations of their inputs, which are p and q respectively. Therefore, the outputs are ~ p and ~ q. Write these on the outputs of the inverters:



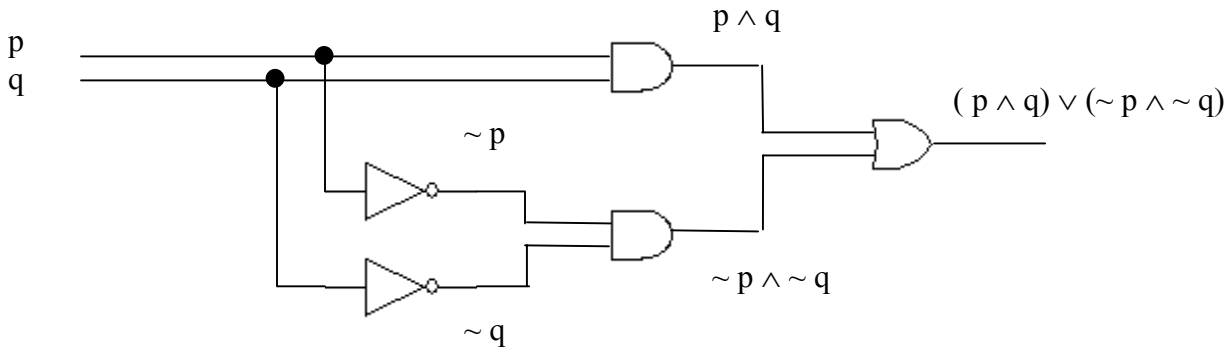
Next, look at the upper AND gate. Its inputs are p and q, so its output is $p \wedge q$. Label its output with this formula.



Next, do the same for the lower AND gate. Its output is ~ p ∧ ~ q.



Finally, the OR gate. Its inputs are (p ∧ q) and (~ p ∧ ~ q), so its output is
(p ∧ q) ∨ (~ p ∧ ~ q).

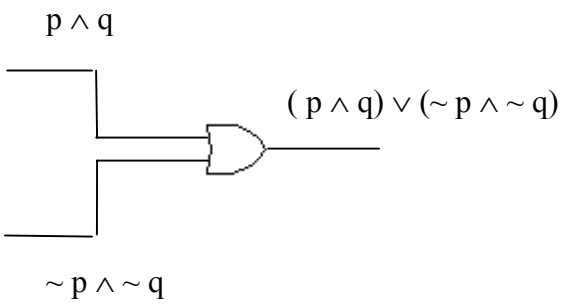


On the next page, we'll see how to do this process in reverse, starting with a formula.

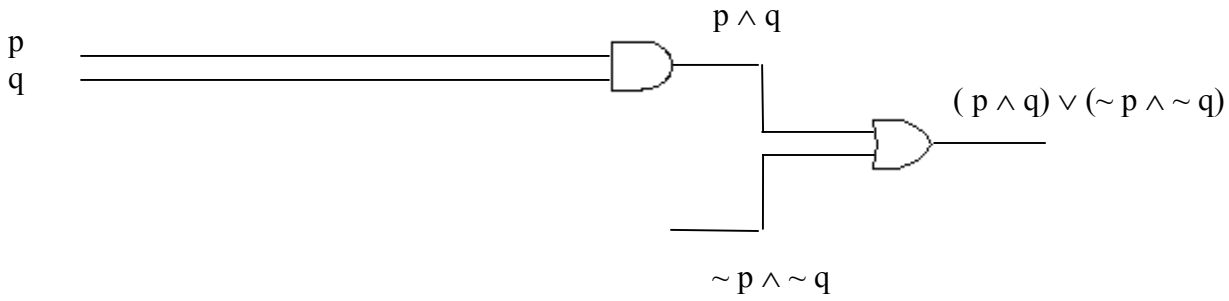
Suppose we are given the formula $(p \wedge q) \vee (\sim p \wedge \sim q)$. We need to locate the main connective (the one that is performed last), and its inputs. The main connective is $\vee$. The following copy shows the main connective and its inputs:

$(p \wedge q) \vee (\sim p \wedge \sim q)$.

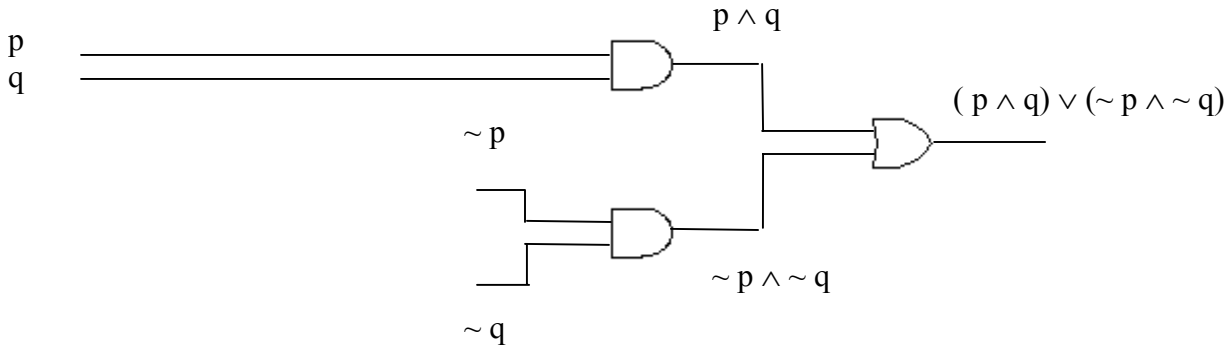
Draw an OR gate, and label its inputs with the formulas highlighted in yellow:



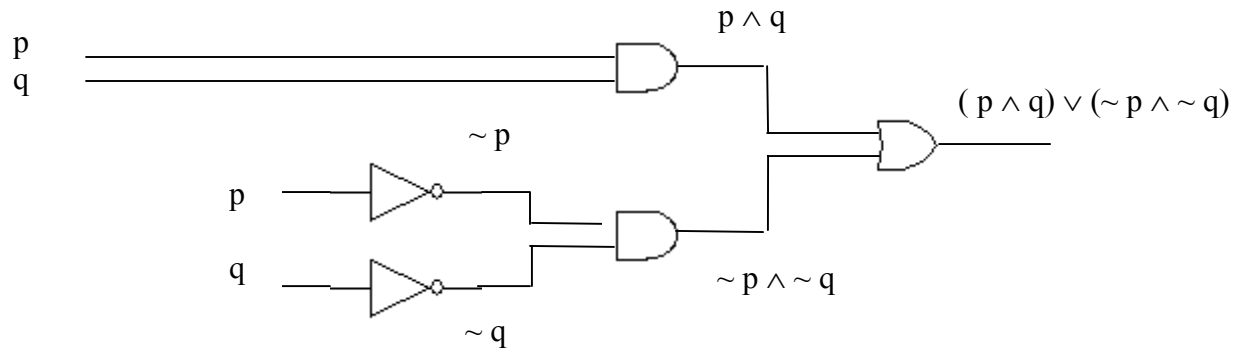
Now we continue to work from right to left. The formula $p \wedge q$ performs and AND operation on two inputs p and q. Draw an AND gate and label its inputs:



The formula $\sim p \wedge \sim q$ performs an AND operation on two inputs $\sim p$ and $\sim q$. Draw an AND gate and label its inputs:



The formulas ~ p and ~ q indicate that two inverters are required; the inputs to the two inverters are p and q. Draw the inverters and label their inputs:



At this point we have a complete diagram, but we can present it better by connecting the two inputs labeled p and the two labeled q. The final result is